Exam Nuclear&Hadron Physics, 26/06/2012

- Write your **name** and **student ID** on each sheet.
- Pay attention to units. A numerical result without a unit will be considered wrong!
- Only a regular calculator is allowed.
- This is NOT an open book exam.
- You are allowed to bring one A4 page with your own notes (one side only).
- You have **3 hours** to complete the exam.

Problem 1: General (3 points)

Briefly discuss the following general questions.

a) Describe the main features of the *Parton model* for deep-inelastic scattering. What is the *Callan-Gross relation* and what can one learn from it?

b) Explain the concept behind a *Partial Wave Analysis (PWA)*. Why is it difficult to apply such a method to the Coulomb interaction?

c) The *Electric Dipole Moment (EDM)* of nuclei is practically zero. Explain why this is the case.

Problem 2: Charmonium (3 points)

Charmonium consists of a charm quark and a charm anti-quark ($c\bar{c}$). The ground state of charmonium is called the η_c state which has a resonance mass of M=2980 MeV/c^2 and a total width of Γ=28 MeV. The spin-parity of the η_c is J^{PC}=0^{-+}.

a) Which orbital angular momenta between the quark and anti-quark are allowed for the η_c state? Motivate your answer.

b) The η_c can be studied via the annihilation of a beam of anti-protons on a proton target. Consider the reaction $\bar{p}+p \to \eta_c(2980) \to \bar{p}+p$. Calculate the anti-proton beam energy that is needed to populate the η_c at its resonance mass.

c) The branching fraction B($\eta_c \to \bar{p}p$)=1.3×10^{-3}. Calculate the total cross section of the reaction $\bar{p}+p \to \eta_c(2980) \to \bar{p}+p$ at a center-of-mass energy of $\sqrt{s}$=2980 MeV.

Problem 3: Partial wave decomposition (2 points)

The scattering amplitude of a scattering process can be described by a partial-wave decomposition as

$$f(\theta) = \frac{1}{k}\left[ie^{-ka}\sinh(ka) + 3e^{ika}\sin(ka)\cos\theta\right],$$

with the wavenumber k of the incident particles, the scattering angle θ, and a real parameter a with dimension [fm].

a) Find an expression for the ℓ=0 and ℓ=1 phase shifts, δ_0 and δ_1, in terms of k and a. Draw the corresponding Argand diagrams for $\eta_\ell(k) = e^{2i\delta_\ell}$.

b) What is the total, elastic plus inelastic, cross section?

Problem 4: Elastic proton-proton scattering (2 points + 1 bonus)

The one-pion-exchange potential, V_{OPE}, for elastic proton-proton scattering without a tensor force is given by

$$V_{\mathrm{OPE}} = \frac{4}{3}\pi f_{\pi NN}^2 \hbar c(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\frac{e^{-kr}}{r},$$

with $k = m_\pi c/\hbar$ and $f_{\pi NN}^2 = 0.0755$. The operators $\vec{\sigma}_{1,2}$ represent the Pauli matrices for the spin of proton 1 and 2 and $\tau_{1,2}$ ($\vec{S}_{1,2}/\hbar = \vec{\sigma}_{1,2}/2$) the Pauli matrices for the isospin of proton 1 and 2 ($\vec{I}_{1,2} = \vec{\tau}_{1,2}/2$), respectively.

a) Show that for S-wave proton-proton scattering, the expectation values of $\vec{\tau}_1 \cdot \vec{\tau}_2 = +1$ and $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3$.

b) Obtain an expression for the scattering amplitude in first-order Born approximation using above potential with $\vec{\tau}_1 \cdot \vec{\tau}_2 = +1$ and $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3$. Calculate the corresponding differential cross section, $d\sigma/d\Omega$, for a proton with a kinetic energy of 100 MeV and scattered at an angle of 20°, both in the center-of-mass frame.

c) Find an expression of the ℓ=0 phase shift as a function of the incident momentum for the potential given in b). You may assume that the phase shift is small. [Hint: make use of the orthogonality relation of Legendre polynomials: $\int_\Omega P_\ell P_{\ell'} d\Omega = 4\pi\delta_{\ell\ell'}/(2\ell+1)$.]

List of expressions, constants, etc.

- $\hbar c = 197$ MeV fm.
- fine structure constant $\alpha = 1/137$.
- mass of the (anti)proton: $m_p = 940$ MeV/c^2.
- mass of the pion: $m_\pi = 140$ MeV/c^2.
- Legendre polynomials P_ℓ: $P_0 = 1$, $P_1 = \cos\theta$, $P_2 = \frac{1}{2}(3\cos^2\theta - 1)$.
- Commutation relation for Pauli spin matrices $\vec{\sigma}$: $\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$.
- $\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$.
- $\sinh(x) = (e^x - e^{-x})/2$.